Optimal Power Flow in Presence of Uncertainties

Ai-Chih Lin, Shin-Yeu Lin

1Department of Electrical Engineering, Chang Gung University, Taoyuan, Taiwan
2Department of Electrical Engineering & Green technology Research Center, Chang Gung University, Taoyuan, Taiwan

1m0021013@stmail.cgu.edu.tw, 2shinylin@mail.cgu.edu.tw

Abstract—This paper presents a new algorithm for the optimal power flow (OPF) in the presence of uncertainties caused by renewable energy sources. The proposed algorithm consists of two parts. One part is the off-line analysis to identify the probability distribution model of the concerned security terms. The other part is the on-line application algorithm to determine the restricted upper and lower bounds of the security terms. We test the proposed algorithm on the OPF problem in the presence of uncertainties of a 26-bus system. The test results demonstrate that the proposed algorithm is efficient and effective.

I. INTRODUCTION

In recent years, global warming is a serious problem that needs to be dealt with in the current world [1-2]. To reduce the global warming effect, we need to reduce the gas emissions. Since non-renewable electricity utilization accounts for about 24% of greenhouse gas emissions [3], a fundamental transformation in the way electricity is generated, delivered, and utilized must take place so as to “decarbonize” the power sector. For example, coal-based generation needs to be reduced, and the nuclear and renewable power generation must make bigger contributions.

Global tendency toward environmental-friendly energy sources with lower generation costs led to the increasing penetration of renewable energies in power systems [4, 5]. Most renewable energy sources are highly intermittent; they will induce significant fluctuation on the supply side of the power grid and cause significant uncertainties on the demand side as well. The smart grid is a modernized and upgraded electricity grid that uses the most advanced information and communication technologies to optimize its operation and to fully accommodate numerous distributed renewable power generation [6, 7]. Therefore, it is a challenging task to guarantee that power demand and power generation remains balanced under the intermittency of power generation [8, 9].

Under the uncertainties of the power generated by the renewable energy sources, the conventional optimal power flow (OPF) problem requires some new formulation. Taking the variation of renewable energy generation into account, the security constraints in the OPF problem need to be reconsidered so as not to exceed the real security limits. This is the main theme of this paper.

We organize our paper as follows. The OPF problem in the presence of uncertainties is formulated in Section II. The proposed solution algorithm is presented in Section III. The test results of the proposed algorithm applied to solve the OPF problem in the presence of uncertainties on a 26-bus test system are presented in Section IV. Finally, the conclusion and further research are drawn in Section V.

II. PROBLEM FORMULATION

The smart grid will make full use of the power generated by renewable energy sources; therefore we only consider the total cost of non-renewable-energy power generation buses. We assume that there are \( N \) buses and \( m \) security constraints in the system. The formulation for the OPF problem in the presence of uncertainties can be stated in the follows:

\[
\begin{align*}
\min & \quad \sum_{i \in G, G} a_i P_{Gi}^2 + b_i P_{Gi} + c_i \\
\text{subject to} & \quad g_i(x, P_G, Q_G, P_r, Q_r, P_D, Q_D, Y) = 0, \quad i = 1, \ldots, N \\
& \quad P_{G_{min}} \leq P_G \leq P_{G_{max}}, \quad Q_{G_{min}} \leq Q_G \leq Q_{G_{max}}, \quad i = 1, \ldots, L \\
& \quad \min_{j \in (1, \ldots, m)} \left\{ h_j^L \leq h_j(x, Y) \leq h_j^U \right\} \geq \delta
\end{align*}
\]

where \( g_i(x, P_G, Q_G, P_r, Q_r, P_D, Q_D, Y) = 0 \) denotes power balance equation of the \( i \)th bus; \( x \) denotes the vector of bus voltage magnitude and phase angle; \( P_G \) and \( Q_G \) denote the vector of real and reactive power generation of non-renewable-energy power generation bus, respectively; \( P_r \) and \( Q_r \) denote the vector of real and reactive power generation of renewable-energy sources; \( P_D \) and \( Q_D \) denote the vector of real and reactive power demand of load bus; \( Y \) denotes the admittance matrix of the transmission lines of the system; \( P_{G_{min}} \) and \( P_{G_{max}} \) denote the lower and upper bounds of the real power generation of non-renewable-energy power generation bus; \( Q_{G_{min}} \) and \( Q_{G_{max}} \) denote the lower and upper bounds of the reactive power of the non-renewable-energy power generation bus; \( h_j(x, Y) \) is the concerned security term; \( h_j^L, h_j^U \) denote the actual lower and upper bounds of the security limits. Notably, \( P_r \) and \( Q_r \) are random variables. Under the uncertainties of the power generated by the renewable energy sources, \( h_j(x, Y) \) is also a random variable.

Therefore, the new formulation of the security constraints

\[
\min_{j \in (1, \ldots, m)} \quad \text{Prob} \left\{ h_j^L \leq h_j(x, Y) \leq h_j^U \right\} \geq \delta
\]

is intended to keep the concerned security term \( h_j(x, Y) \) within security limits with probability more than \( \delta \). Generally, the closer \( \delta \) to
1, the better. If $\delta = 1$, we do not need any power generation reserve.

To cope with probabilistic security constraint is a challenging task, which makes the proposed algorithm to be presented in the following section distinguishable.

$$P_{h_j(x,Y)} \text{ Prob } \left\{ \bar{z}_{j\delta} \leq h_j(x,Y) \leq \bar{z}_{j\delta} \right\} = \delta$$

Fig. 1. Schematic diagram of $\mu_j$, $\bar{z}_{j\delta}$, $\bar{z}_{j\delta}$.

III. PROPOSED SOLUTION ALGORITHM

To tackle the OPF problem in the presence of uncertainties (1), we proposed a solution algorithm, which consists of two parts. One part is the off-line analysis of the probability distribution model of the concerned security terms; the other is the on-line application algorithm. The reason for the off-line study to establish the probability distribution model of the concerned security terms is that the Monte Carlo simulations is too time consuming to be used on-line.

A. Off-Line Analysis

The uncertainties of the power generated by the renewable energy sources causes the uncertainties of the concerned security terms. The analysis of the probability distribution model of the concerned security terms should be based on the probability distribution model of the power generated by the renewable energy sources.

For the sake of simplicity in illustration, we use wind power generation as an example of the renewable energy source. We proceed with the off-line analysis of the probability distribution model of the concerned security terms as follows.

1) Randomly generate the wind speed $v$ based on a given Weibull probability density function presented in (2) [10] for each wind power generation bus.

$$p(v) = \frac{\beta}{\alpha} \left( \frac{v}{\alpha} \right)^{\beta-1} e^{-\left( \frac{v}{\alpha} \right)^\beta}$$

where $p(v)$ denotes the probability density function of the wind speed $v$; $\alpha$ and $\beta$ represent the scale and shape coefficients, respectively [11-13].

2) Calculate the generated wind power from the wind speed $v$ obtained in Step 1 for each wind power generation bus by the following formula [13]:

$$P_r = 0.5C_p\rho v^3 A \ (W)$$

where $C_p$ denotes the power coefficient, $\rho$ denotes the air density and $A$ denotes the cross-section swept by the wind turbine blades in $m^2$ ($A = \pi l^2_{\text{blade}}$).

In this off-line analysis, the values of the parameters $\alpha$, $\beta$, $\rho$, $A$ and $C_p$ used in this paper are listed in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ (m/s)</td>
<td>9</td>
</tr>
<tr>
<td>$\beta$ (m/s)</td>
<td>1.6</td>
</tr>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>1.225</td>
</tr>
<tr>
<td>$A$ (m$^2$)</td>
<td>706.8</td>
</tr>
<tr>
<td>$C_p$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

3) Assuming nominal values of $P_G$, $Q_G$, $P_D$, $Q_D$, $Y$ and the values of $P_r$ and $Q_r$ obtained from Step 2, we perform a power flow analysis.

4) Calculate $h_i(x,Y)$, $j = 1, \ldots, m$ from the power flow solution in obtain in Step 3.

5) Repeat Steps 1 to 4 for 10000 random samples.

6) Using the model fitting method to find the probability distribution model for each $h_i(x,Y)$ based on the obtained 10000 random samples.

The obtained probability distribution model for each $h_i(x,Y)$ will be used in the on-line application algorithm.

B. On-Line Application Algorithm

The on-line application algorithm is designed to determine the lower and upper limits of security constraint denoted by $h_j^L$ and $h_j^U$ respectively, shown in (4)

$$\begin{align*}
\min & \sum_{i=1}^n a_i P_{Di} + b_i P_{Ci} + e_i \\
\text{subject to} & \ g_i(x, P_G, Q_G, P_D, Q_D, Y) = 0, \ i = 1, \ldots, N \\
& P_{D_i \min} \leq P_{D_i} \leq P_{D_i \max}, \ Q_{D_i \min} \leq Q_{D_i} \leq Q_{D_i \max}, \ i = 1, \ldots, L \\
& h_j^L \leq h_j(x,Y) \leq h_j^U, \ j = 1, \ldots, m
\end{align*}$$

so as to obtain the solution of (1).

The basic idea of our approach can be stated in the following. First, we calculate the expected wind speed and the corresponding wind power generation based on the given wind speed probability distribution. Setting $\bar{P}_r$ and $\bar{Q}_r$ as the obtained expected wind power generation, then, initially guess the lower and upper bounds of security constraints $h_j^L$ and $h_j^U$ and solve (4) using the OPF method. Let $P_{r_0}^*$ and $Q_{r_0}^*$ be the obtained optimal solution of non-renewable-energy generation buses. Setting the power generation of non-renewable-energy generation buses at $P_{r_0}^*$ and $Q_{r_0}^*$, treating $P_r$ and $Q_r$ as random variables, we then use $2k + 1$ point estimation method to estimate the mean $\mu_j$ and standard division $\sigma_j$ of the concerned security term $h_j(x,Y)$. Based on the obtained $\mu_j$ and $\sigma_j$ of the concerned security term $h_j(x,Y)$, we can obtain the parameters $\alpha_j$ and $\beta_j$ of the Beta probability density function derived from the off-line analysis for $h_j(x,Y)$ by (5) and (6).

$$\mu_j = \frac{\alpha_j}{\alpha_j + \beta_j}$$

$$\sigma_j = \left( \frac{\alpha_j \beta_j}{(\alpha_j + \beta_j)^2(\alpha_j + \beta_j + 1)} \right)$$
Then, we can obtain the corresponding $z_{j\delta}$, $\bar{z}_{j\delta}$ for $h_j(x, Y)$ using the obtained Beta probability density function shown in (7)

$$f(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

(7)

Notably, the values of $z_{j\delta}$, $\bar{z}_{j\delta}$ will be used to update the value of $h_j^s$ and $\bar{h}_j^s$ in the on-line application algorithm.

Now, we can state the algorithmic steps of the on-line application algorithm in the following.

0) Calculate $P_r$ and $Q_r$ and set $h_j^s = h_j^o$ and $\bar{h}_j^s = \bar{h}_j^o$ for all $j = 1, \ldots, m$.

1) Solve the OPF problem (4) and obtain the optimal $P_G^{*}$ and $Q_G^{*}$ for non-renewable-energy generation buses.

2) Setting $P_T = P_T^*$ and $Q_T = Q_T^*$ for the non-renewable-energy generation buses, treating $P_r$ and $Q_r$ as random variables, then use 2$k$ + 1 point estimation method [14] to solve the following probabilistic power flow problem:

$$g_i(x, P_G^*, Q_G^*, P_r, Q_r, P_D, Q_D, Y) = 0, \quad i = 1, \ldots, N$$

(8) and estimate the $\mu_j$ and $\sigma_j$ of $h_j(x, Y)$.

3) Input $\mu_j$ and $\sigma_j$ into (5) and (6) to obtain the parameters $\alpha_j$ and $\beta_j$, then use $\alpha_j$ and $\beta_j$ to find the corresponding $z_{j\delta}$, $\bar{z}_{j\delta}$ based on the probability distribution given in (7).

4) If there is no $j$ such that $z_{j\delta} \leq \bar{h}_j^o$ and $z_{j\delta} \geq h_j^o$, go to Step 5. Otherwise, if $z_{j\delta} > \bar{h}_j^o$, set $h_j := h_j^o - (z_{j\delta} - \bar{h}_j^o)$; if $z_{j\delta} < h_j^o$, set $h_j := h_j^o + (h_j^o - z_{j\delta})$; then return to Step 1.

5) Output the solution $P_G^{*}$, $Q_G^{*}$.

IV. SIMULATION RESULTS

We test the proposed algorithm on a 26-bus system [15]. In this test system, we assume that buses 4 and 5 are wind power generation buses, while buses 1, 2, 3 and 26 are non-renewable-energy generation buses. We assume that the operation cost of non-renewable-energy generation bus is $a_i P_G^{*} + b_i Q_G^{*} + c_i$. Table II presents the operation cost coefficients $a_i$, $b_i$, and $c_i$ of non-renewable-energy generation buses.

The Weibull probability distribution for wind speed is presented in Fig. 2. We perform the off-line analysis to obtain the corresponding probability distribution model of all concerned security terms. Fig. 3 presents the probability distribution of the voltage magnitude of a typical bus, say bus 9, obtained from the off-line analysis. The probability distribution of a typical line flow, say line 9-10, obtained from the off-line analysis is presented in Fig. 4. From both Figs. 3 and 4, we can observe that they are Beta-distribution.

The on-line application algorithm took five iterations to determine the values of $h_j^s$ and $\bar{h}_j^s$ for all $j$. The algorithm is implemented in MATLAB code and carried out in an Intel Core i5 3.3-GHz PC with 4GB of RAM.

The resulted $P_G^{*}$, $Q_G^{*}$ for buses 1, 2, 3 and 26 are presented in Table III, and the consumed CPU time is 0.174957 seconds.

<table>
<thead>
<tr>
<th>Bus</th>
<th>$P_G^{*}$ (MW)</th>
<th>$Q_G^{*}$ (Mvar)</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>477.836</td>
<td>259.932</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>195.786</td>
<td>58.766</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>286.882</td>
<td>78.262</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>110.551</td>
<td>27.758</td>
<td></td>
</tr>
</tbody>
</table>

TABLE III
THE ON-LINE APPLICATION ALGORITHM RESULTS OF THE NON-RENEWABLE-ENERGY GENERATION BUSES

Fig. 2. Weibull probability distribution of the wind speed based on the parameters given in Table I.

Fig. 3. Bus voltage of the probabilistic power flow from the wind speed distribution shown in Fig. 2.
Fig. 4. Line flow of the probabilistic power flow from the wind speed distribution shown in Fig. 2.

V. CONCLUSIONS
We have presented an algorithm to tackle the OPF problem in the presence of uncertainties caused by the renewable energy sources. The proposed algorithm consists two parts, the off-line analysis and the on-line application algorithm. The on-line application algorithm includes a point estimate method to estimate the mean and variance of the concerned security terms, so as to identify its probability distribution model. We have successfully applied the proposed algorithm on a 26-bus test system, and the results demonstrate that the proposed algorithm is efficient and effective.

REFERENCES