This paper investigates the implementation of a predictive controller using nonlinear dynamic model derived from first principles, together with online optimization, for a rotary crane system. The system is driven by two motors in velocity mode, and a white LED is installed on the payload to be observed by a camera located on the rotary frame to derive swing angles for control feedback. The proposed prediction controller is featured with control horizon long enough for online computation of prediction and optimization algorithms. Possible control sequences within the control horizon are described using spline interpolation to reduce the number of parameters to be optimized online. Response of the proposed scheme in face of exogenous disturbances and inaccuracy in estimated string length is demonstrated by numerical simulations.

KEY WORDS
Sway Suppression; Model Predictive Control; Online Optimization.

1. Introduction

Rotary cranes are important carriers in building construction, factories, and harbors. These cranes are subject to dangerous swing when fast motion is expected. To enhance safety in these working conditions, automatic control of the cranes is preferable.

Many researches focused on the model-based anti-swing control of overhead cranes, such as the feedforward strategy proposed in [3], the Sliding model control [7], neural network control [2], the fuzzy logic controller [1][4][8], and the "soft sensor" in [9] to replace expensive sensors.

Considering the difficulty in installing durable encoders at the hanging base of string, we install a white LED on the payload and a camera on the rotary frame to derive swing angles for control feedback. The approach is similar to [13]. Besides, a mechanism for axial motion is provided for extra manipulation capability.

The motors in the system are controlled in velocity mode as most industrial standard ones. This assumption also simplifies nonlinear effects caused by friction and load effects. In doing so, system bandwidth will be significantly reduced when compared with torque mode. The situation is further deteriorated by the time required for visual servo. To compensate for these time-delay effects, a robust predictive controller is proposed.

In order to reduce the number of parameters for on-line optimization, possible control sequences within control horizon are encoded using cubic spline interpolation. Robustness of the proposed scheme is demonstrated by simulation of the strategy under external disturbances and model inaccuracy.

2. Modelling of the Rotary Crane System

A rotary crane contains an arm, also called the jib, which rotates in a horizontal plane around a fixed vertical axis. The load is attached to the arm using a cable. Figure 1 shows the crane model.
that holds the cable. The distance between the origin of frame \{Q\} and the \(Z_1\) axis of frame \{O\} is \(r\). Besides, a moving coordinate system \{S\} is attached to the plane containing the load, which is parallel to the \(X_1Y_1\)-plane. The origin of \{S\} is right below that of \{Q\} while the second axis of it, the \(V\)-axis, is parallel to the jib.

Under the assumption that the cable can be simplified as a straight line, the load positions are uniquely described by three angles, \(\alpha\), \(\theta\) and \(\phi\), where \(\alpha\) is the angle that the jig makes with the \(X_1\)-axis in the \(X_1Y_1\)-plane, \(\phi\) is the angle that the cable makes with the \(W\)-axis, and \(\theta\) is the angle that the cable makes with the \(U\)-axis in the \(UV\)-plane.

The objective of the controller is to move the payload while keeping the load track a trajectory described in the fixed coordinate system \{O\} without swinging back and forth. Taking moment about the origin of frame \{Q\} along the \(U\), \(V\), and \(W\) axes, we have

\[
-m \cdot g \cdot p \cdot \sin \theta = m \cdot \dot{v} \cdot l \cdot \cos \phi + m \cdot \ddot{z} \cdot v_L
\]

(1)

\[
m \cdot g \cdot p \cdot \cos \theta = -m \cdot \ddot{u} \cdot l \cdot \cos \phi - m \cdot \ddot{z} \cdot u_L
\]

(2)

\[0 = -m \cdot \ddot{u} \cdot v_L + m \cdot \ddot{v} \cdot u_L
\]

(3)

where \(p\) is the distance between the load and the origin of frame \{S\}, \(u_L\), \(v_L\) are the velocity of the load in the direction of axis \(U\) and \(V\), and \(\ddot{u}\), \(\ddot{v}\), and \(\ddot{z}\) are the acceleration of the load in the direction of axis \(U\), \(V\), and \(W\), respectively.

Taking coordinate transformation between static coordinate \{O\} and moving coordinate system \{S\}, we have

\[
\dot{\theta} = -\frac{1}{l \sin \phi} \left[ -l \ddot{u} \sin \phi - 2l \phi \dot{u} \cos \phi + r \dot{\alpha}^2 \cos \theta \right]
\]

(4)

\[-2r \alpha \dot{\alpha} \sin \theta - r \alpha \ddot{\alpha} \sin \theta - \ddot{r} \cos \theta - 2l \phi \dot{\phi} \cos \phi \right] + b_1 \dot{\theta}
\]

\[
\quad + \frac{1}{l} \left[ -g \sin \phi \right] + \left[ -r \sin \theta \dot{\cos \phi} + 2l \theta \dot{\alpha} \cos \phi \sin \phi \right. 
\]

\[
\left. + l \dot{\theta}^2 \cos \phi \sin \phi + l \alpha^2 \cos \phi \sin \phi + r \dot{\alpha}^2 \sin \theta \cos \phi \right] + b_2 \dot{\phi}
\]

(5)

where \(b_1\) and \(b_2\) are damping coefficients to be decided.

Equations (4) and (5) are in the standard state space form:

\[
\begin{align*}
\dot{x}(t) &= f_p(x(t), u(t)) \\
y(t) &= g_p(x(t)),
\end{align*}
\]

(6)

where the state vector \(x = [\theta, \dot{\theta}, \phi, \dot{\phi}]^T\), the input vector \(u = [\alpha, \dot{\alpha}, \dot{r}, \ddot{r}]^T\), and the output vector \(y\) is defined in the fixed coordinate system \{O\}, such that \(y = [\alpha_g, \gamma_g]^T\), as shown in Fig. 2

An experimental platform is shown in Fig. 3, where a servomotor, HC-KFS43G1 of Mitsubishi Motors, drives the tower of the crane. A second servomotor, HC-KFS43K of Mitsubishi Motors, is connected to a four-bar linkage, which moves the hanging point that holds the cable in the radial direction. Besides, we installed a white LED on the payload and a camera on the jig to derive swing angles for control feedback.

To verify the consistency of the model with the physical system, the crane was driven by the first motor to rotate 90 degrees in 6 seconds. Fig. 4 shows the comparison of the experimental locations of the load and simulated trajectories with prediction horizon of 0.5 sec without the effect of damping.

The model was then modified by the addition of damping terms by trial and error. The damping coefficient \(b_1\) was identified as a function of the angular velocity of the first motor, \(\dot{\theta}\), as shown in Fig. 5. After this modification, the simulated trajectories match that of the experimental results, as shown in Fig. 6.

The correctness of the model is further demonstrated by a comparison of trajectories moving 120 degrees in 6 seconds, as shown in Fig. 7.

In Figures 4 to 7, the predicted trajectories are shown every 0.5 seconds, which is the length of the prediction horizon of the MPC. In real-time implementation, the control is updated every 0.2 seconds.
3. Model Predictive Control for Rotary Crane System

This section describes the predictive control algorithm investigated. During the time interval between \( t \) and \( t + T_C \), the controller simulates a dynamic model described by:

\[
\begin{align*}
\dot{x}(\tau) &= \hat{f}(\hat{x}(\tau), u^*(\tau)), \quad t + T_C \leq \tau \leq t + T_C + T_P \\
\hat{y}(\tau) &= \hat{g}(\hat{x}(\tau))
\end{align*}
\]  

using estimated initial state vector \( \hat{x}(t+T_C) \) and trial control sequences \( u^* \), where \( \hat{f}(\cdot) \) and \( \hat{g}(\cdot) \) are functions of the plant model, \( T_P \) is the prediction horizon, and \( y^* \) is the desired output function. Relations between these parameters are shown in Fig. 8. Note that \( \hat{x}(t+T_C) \) is itself obtained by the simulation of (7) using control input sequence decided in the last time interval between \( t \) and \( t-T_C \).
For each trial, taking $u_i^*$ for $i$-th trial as an example, there is a cost value defined by the weighted sum of tracking errors between the desired trajectory $y_d(\tau)$ and the corresponding predicted output $\hat{y}_i(\tau)$,

$$Cost_i = \int_{t_i}^{t_i+T_p} \|y_d(\tau) - \hat{y}_i(\tau)\| \, d\tau + \sigma \int_{t_i}^{t_i+T_p} \|u_i^*(\tau)\| \, d\tau$$  \hspace{1cm} (8)

where $\sigma$ is a weighting factor, which is set as 1 in the following simulation.

With the weighting of (8), the best control performance is shown in Fig. 9 and 10, where the overshoot is amount to 158° in the $\alpha_g$ direction. To improve the performance, different weights can be applied in the two orthogonal directions, $\alpha$ and $r$, and in different stages.

In doing so, the cost function is modified as:

$$Cost^* = \psi \cdot F_\alpha + \zeta \cdot F_\gamma$$  \hspace{1cm} (9)

where $\psi$ and $\zeta$ are weights for the sub-cost functions defined in the following equations.

$$F_\alpha = \int_{t_i}^{t_i+T_p} \|\alpha_d(\tau) - \hat{\alpha}_g(\tau)\| \, d\tau + \sigma \int_{t_i}^{t_i+T_p} \|\hat{\alpha}(\tau)\| \, d\tau$$

$$F_\gamma = \int_{t_i}^{t_i+T_p} \|\gamma_d(\tau) - \hat{\gamma}_g(\tau)\| \, d\tau + \sigma \int_{t_i}^{t_i+T_p} \|\hat{\gamma}(\tau)\| \, d\tau$$

After many trial-and errors, the best performance corresponding to the situation when the weightings, $\psi$ and $\zeta$, are set to different values in different stages, as detailed in Fig. 11, where the value of $\zeta$ increases to 10 at the final stage when the load approaches its final destination. The tracking performance of the system with the modified weights is shown in Fig. 12 and 13. We have that the performance improves dramatically when compared with Fig. 9 and 10.
To further reduce the complexity of the optimization procedure, the candidate control sequences are sampled from cubic splines defined by a few parameters. As shown in Fig. 14, the splines are defined by only three parameters, \(h_1\), \(h_2\), and \(h_3\), which describe the control points located at \(t + T_C\), \(t + 0.5T_C\), and \(t + 2T_C\), respectively. That is, each trial control sequence is encoded into 3 parameters which define a cubic spline. The decoding procedure is simply to sample the spline in predefined interval.

Furthermore, although the motors are controlled in velocity mode, the angular acceleration of them is fixed at pre-specified values. The restriction is common to most of the industrial servo motors. The control sequences are then approximated according to this restriction so that angular velocities are constant at most periods and accelerate only at the sampling instants with various durations, as shown in the example of Fig. 15.

The problem is then to find the best control sequence such that the cost is minimized. In order to find the best solution online, the Nelder–Mead method or downhill simplex method [10] is used. Considering that the method suffers from the possibility of being trapped in local minima, we actually start four polytopes of 4 vertices in 3 dimensions randomly for 20 iterations, and then start a polytope with the 4 best results of these polytopes as vertices and run for 20 more iterations. A typical example is shown in Fig. 16.

4. Simulation Investigation

Several simulations were conducted to investigate the performance of the predictive control strategy under various setting of design parameters, and robustness with respect to modeling discrepancy and external disturbance.

In the following simulation study, the payload is assumed to be 2.851 kg hanging from a cable of 920 mm with nominal jig length being 310 mm, and the control signal is updated with a sampling period of 0.05 sec, according to the experimental prototype, and \(T_P = 10 T_C\). The reference trajectory is to carry the load under an angular translation of 90 degree in 3 seconds without vibration in the radial direction, as shown in Fig. 17.

4.1 Tolerable Control Horizon

In the prediction control, control horizon \(T_C\) determines available online computation time for simulation as well as optimization. Longer \(T_C\) alleviates the demand on computing power of the controller. Fig. 18 and 19 show the tracking performance of the system for several setting of control horizon ranging from 0.1 sec to 0.6 sec. It is clear that the control horizon is allowed to be prolonged to 0.3 sec without inducing significant vibration. The corresponding control signals, angular velocities of the motors are shown in Fig. 20 and Fig. 21 when the control horizon is 0.3 sec.
4.2 Robustness with Respect to Inaccuracy in Cable Length

Simulations to study the effect of discrepancy between the model in the predictive controller and the plant are also conducted. Fig. 22 and 23 show the tracking performance when the cable is modeled in different lengths while the actual length is 0.31 m. According to the results, we have that the performance is acceptable when the model length is within 0.175 m and 0.4 m, a wide tolerable range.

4.3 Response to Disturbance

The effect of disturbance is studied by assuming an extra velocity of 57.3 deg/sec in the \( \theta \) direction during the period of 1.5 to 2.0 sec. Tracking performance of the predictive controller in Fig. 24 and 25 show that the performance is still acceptable.

Fig. 26 and 27 show the responses when disturbances of different directions are applied in the period between 5.5 to 6.5 sec. These results clearly show the effectiveness of the control strategy.
5. Conclusion

This paper investigates the predictive control of a rotary crane system by online optimization and direct application of nonlinear dynamic model. The control sequences are described by cubic splines to reduce the number of design parameters to accelerate optimization computation. Response of the system in the face of modeling error and external disturbance is demonstrated by simulation results. Future research will be focused on online adaptation of the predictive control law and experimental investigation.

References


