A consignment inventory model for deteriorating items with stock-dependent demand and space constraint

Abstract

"Consignment inventory" is the process that the supplier places goods at the customer’s location without receiving payment until after the goods are used or sold and it can be mutually beneficial to both parties. According to previous studies, displaying numerous goods on the shelf might stimulate customer demand. However, stocking too much inventory may tie up retailer’s capital and contradicts with the reality that shelf space is usually not enough.

This thesis considers the consignment inventory problem where a single supplier produces a single deteriorating item for a single retailer. Demand rate at the retailer’s side is dependent on the display inventory subject to the space limit. This study builds an inventory model and finds the replenishment policy for both parties to maximize their integrated profit in the supply chain.

Sensitivity analyses on the parameters conclude as the follows. First, the integrated profit...
increases as the retailer’s shelf space increases; but the profit remains the same once the space exceeds a ceiling. Second, as the coefficient of demand versus inventory increases, then the supplier’s replenishment frequency, retailer’s order quantity and their joint profit increase. Third, the joint profit is a decreasing function of the deterioration rate; while taking into account the retailer’s space constraint, the marginal loss of the profit increases as the deterioration rate does.

Keywords: stock-dependent demand, consignment inventory, space constraint, deteriorating items

一、Introduction

Inventory model with stock-dependent-demand rate in inventory theory is a subject that has recently received considerable attention. These models assume retail inventory to have a motivating effect on the customer. For example, Whitin (1957) stated, “For retail stores, the inventory control problem for style goods is further complicated by the fact that inventory and sales are not independent of one another. An increase in inventories may bring about increased sales of some items. “Wolfe (1968) presented empirical evidence of this relationship, noting that the sales of style merchandise, such as women’s dresses or sports clothes, are proportional to the amount of inventory displayed. Thus, increased inventory levels give the customer a wider selection and increase the probability of making a sale. However, stocking too much inventory may tie up retailer’s capital. In the inventory control models, traditional EOQ (Economic Order Quantity) and EPQ (Economic Produce Quantity) only considered one side optimal. Both models have some artificial assumptions, such as no deterioration, no shelf space constraint and known demand.

Many researchers have recently given considerable attention to coordinating the issue between suppliers and retailers in the supply chain. However, most researchers assume that the chain’s market demand is either price sensitive or constant. Consequently, this work considers coordinating a single supplier-single retailer distribution system. Consignment inventory is a mechanism of supply chain management. Adopting consignment arrangement companies can dramatically lower company costs and drop prices to increase demand for products, attract new customers and even enter new markets. A consignment arrangement makes no payments to the retailer until the item sells, therefore the retailer has no money tied up in inventory and bears no risk
associated with demand uncertainty. The supplier possesses market condition information directly, mitigating bullwhip effect in the supply chain. This increases firm’s measurement accuracy of the retailers’ selling efforts, reducing demand uncertainty, and enabling the supplier to better match supply with demand.

In practice, many retailer stores, like Costco, IKEA, Wall-Mart, etc. stock large piles of goods on their shelf or floor to attract customers and obtain more sale probability. Many large-scale retailer stores have recently created private brands not owned by a manufacturer or producer, but by a retailer or supplier that obtains its goods from a contract manufacturer under its own label. Private brand is also called private label. Private retailer brands create more pricing freedom and flexibility, more control over product attributes and quality, higher margins (or lower selling price), and eliminate considerable manufacturer promotional costs. The retailer must have powerful bargaining power to attract suppliers to sign a consignment contract. However, a private brand encourages consignment contracts, in that it is applicable to consignment policy, putting the supplier and retailer on the same.

Based on the above, this thesis considers a single-manufacturer, single-buyer supply chain problem under the consignment arrangement. Where retailer demand is stock-dependent, the retailer has shelf space constraint and considers the deterioration issue.

二、Literature review

2.1 Stock-dependent demand

Traditional inventory models have formed under the assumption of constant demand or time-dependent demand. A number of inventory models have recently formed considering demand to be dependent on inventory level. Levin et al. (1972) observed that “large piles” of consumer goods displayed in a supermarket lead customers to buy more. Silver and Peterson (1985) also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Mass displays of items in stores are used as “psychic stock” (Larson and DeMarais (1990)) to stimulate retail sales of some items. Psychic stock is retail display inventory used to simulate demand. The concept
of psychic stock defined as retail display inventory, models by partitioning psychic, cycle and safety stock. Many researchers have extended this concept.

The preliminary inventory model assumes that the demand rate is constant. Gupta and Vrat (1986) first developed a linear inventory model in the stock-dependent-demand rate. Soni and Shah (2008) formulated the mathematical model for an inventory system with stock-dependent demand and delay payment for the retailer in which the supplier offered two progressive credit periods to the retailer to settle his account. Jain et al. (2008) believed inventory should start deteriorating after a certain time period, rather than after immediate stock arrival. They also argued that demand decreases due to aging inventory items and they incorporated two realistic features and partial backlogging with the stock-dependent demand inventory model. Goyal and Chang (2009) formulated an order-transfer inventory model when the amount of display space is limited and the demand rate depends on display stock level. Their objective was to simultaneously determine retailer’s optimal order quantity and number of transfers per order from the warehouse to the display area for maximizing average profit per unit yielded by the retailer. Min and Zhou (2009) developed an inventory model for perishable items with stock-dependent demand and imposed a ceiling on the number of on-display stocks. In the model, unsatisfied demand is partially backlogged and the backlogged demand rate is dependent on the negative inventory level during the stock-out period.

Baker and Urban (1988) developed an inventory model different from Gupta and Vrat (1986) in which the demand rate is an exponential function. Pal et al. (1993) extended the model of Baker and Urban (1988) for perishable products that deteriorate at a constant rate. Goh (1994) discussed the model of Baker and Urban (1988), relaxing the assumption of a constant holding cost. In his model, he employed the holding cost as a nonlinear function of the length of time the item is held in stock, and a nonlinear function of the amount of on-hand inventory. Balkhi and Benkherouf (2004) presented an inventory model for deteriorating items with stock-dependent and time-varying demand rates for a finite time planning horizon. Alfares (2007) considered an inventory policy for an item with stock-level dependent demand rate and storage-time dependent holding cost. The study assumed that holding cost per unit of item is an increasing function of the time spent in storage. Urban (2008) modified Alfares policy (2007) for maximizing profit and allowing nonzero
ending inventory. In the stock-dependent demand model, inventory decisions affect demand rate. Thus, maximizing profits encourages a higher demand rate by maintaining higher inventory levels. Zhou et al. (2008) considered a similar coordination issue of a decentralized two-echelon supply chain. The demand rate is dependent on the inventory-level on display and the supplier provides quantity discounts to attract retailers to increase order quantities.

Datta and Pal (1990) presented an inventory model in which the demand rate is dependent on instantaneous inventory level until achieving a given inventory level $S$, after which the demand rate becomes constant. One of the terminal conditions used in this model is that the cycle ends with zero stock. Later, Urban (1992) relaxed the unnecessary zero ending-inventory at the end of each order cycle as imposed in Datta and Pal (1990), because utilizing higher inventory levels is more profitable in the inventory model of stock-dependent demand rate, resulting in greater demand (Chang, 2004). Similarly, Giri et al. (1996) generalized the Urban (1992) model for constant deteriorating items. The above researches and many related papers on stock-dependent-demand merely consider the supply chain issue.

2.2 Consignment inventory

The APICS dictionary (Blackstone and Cox 2004) defines consignment as “…the process of supplier placing goods at a customer location without receiving payment until after the goods are used or sold”. Consignment inventory can be mutually beneficial to both the supplier and the retailer. In consignment purchasing policy, the buyer provides warehouse space for the supplier to stock, thus allowing the supplier generous savings in inventory carrying cost. The buyer can postpone payment until stocks are actually sold or consumed. This implies that a consignment policy might create a shared benefit for both the supplier and the buyer. Goyal (1977) preliminarily developed a single-supplier single-retailer integrated inventory model. Hill (1997) also extended Goyal’s (1977) model to discuss minimizing total cost of a single buyer and single retailer-consignment-inventory problem. Braglia and Zavanella (2003) first proposed a consignment inventory model for solving joint economic lot size problems. His model is related to the assumption implied in the Hill (1997) model and explores a model allowing the supplier to use the buyer’s warehouse and assume demand rate is constant. To fit real situations, Persona et al. (2005) extended Braglia and Zavanella’s (2003) model and considered deteriorating
items in consignment inventory.

Banerjee (1986) developed a joint economic lot-size model and assumed that the supplier followed a lot-for lot shipment policy with respect to the retailer. Lu (1995) explored a model allowing shipments during production. Hill (2000) proposed optimal two-stage lot sizing and inventory batching policies. Yang and Wee (2003) established an integrated multi-lot-size production inventory model for deteriorating items. Lee and Wang (2008) considered a single-manufacturer, single-buyer supply chain problem in which a manufacturer produced a single item product and periodically delivered it to the buyer on a consignment policy basis. Their work constructed an integrated inventory control model, making joint economic lot size decisions for the manufacturer’s production batch and replenishing the lot subject to consignee’s warehouse-space capacity constraint, to minimize manufacturer’s total cost.

This thesis deals with the problem of determining operating policy for a consignment inventory system in which the demand is dependent on the display stock level, limited shelf space, and product deterioration. The objective is to determine an optimal ordering and transfer schedule, which maximizes average profit. This work first develops the no-shelf-space constraint model and then provides a shelf space constraint model.

三、 Notation and assumptions

The following notations are used:

\( R_s \) : the replenishing cost of supplier per order ;

\( S_s \) : the cost of supplier placing per order ;

\( L_s \) : unit deterioration loss at the supplier’s side, which is the unit selling price to the retailer ;

\( L_b \) : unit deterioration loss at the retailer’s side, which is the unit selling price to the customer ;

\( h_b \) : the unit carrying cost per item in the display area ;
\( h_s \) : the unit carrying cost per item in the supplier warehouse:

\( P_b \) : the unit selling price of the product per unit:

\( I_{b_{\text{max}}} \) : the maximal shelf space of the retailer:

\( W_b \) : the shelf space constraint of the retailer:

\( q \) : the quantity per transfer from the supplier to the display area:

\( t_p \) : the producing time of supplier from 0 to \( q \) unit:

\( t_{p_{\text{total}}} \) : the total producing time of supplier until arrival the upper bound of retailer shelf space:

\( t_q \) : the time of retailer’s max stock level consume \( q \) unit:

\( t_{I_{b_{\text{max}}}} \) : the time of retailer’s stock level consume from max shelf space to zero:

\( t_{W_s} \) : the time of retailer’s stock level consume from upper bound to zero:

\( I_s(t) \) : the inventory level at time \( t \) of the supplier:

\( I_{b}(t) \) : the inventory level at time \( t \) in the display area of the retailer:

\( P \) : production rate of supplier, where \( P \) is a constant:

\( \Theta \) : the product deteriorating rate, where \( 0<\Theta<1 \):

\( D(t) \) : the demand rate at time \( t \). We assume that \( D(t) = \alpha + \beta I_{h}(t) \), where \( \alpha \) and \( \beta \) are non-negative constants, respectively.

\( T \) : retailer receives the first replenishment fill:

\( n \) : number of replenishments in a cycle period:

\( m \) : the number of replenishments from the supplier to the retailer before the retailer hits the max inventory level for the first time:

\( TC \) : the sum cost of supplier and retailer:

\( \pi \) : the function of maximal profit

The following assumptions are adopted:
(1). We considered only one supplier and one retailer.
(2). Shortages are not allowed. We assume production rate is larger than demand rate.
(3). The lead time and the time to transfer items from the supplier to the retailer are zero.
(4). Only supplier pay the replenishment cost.
(5). We assume that supplier transfers items for n times will arrival the max shelf space or upper bound of retailer.
(6). The inventory of retailer is zero at the end of time $T$.

四、No shelf space constraint model

Figure 3.1 display the relationship between supplier and retailer with no shelf space constraint. The cycle time of the supplier and the retailer have the same length $T$. Supplier has to produce one batch $q$ at the beginning and transfer to the retailer, so the beginning time of retailer is postpone time $t_p$. 
4.1 The total cost per cycle time $T$ of supplier

The production rate is $P$ and deteriorating rate is $\theta$. So the differential equation expressing the inventory level at time $t$ can be written as follows:

$$\frac{dI_v(t)}{dt} = P - \theta I_v(t)$$

With the boundary condition $I_v(0) = 0$. Accordingly, the solution is given

$$I_v(t) = \frac{P}{\theta} \left(1 - e^{-\theta t} \right), \quad 0 \leq t \leq t_p$$

With the boundary condition $I_v(t_p) = q$. We get the producing time $t_p$ of $q$ units.
The total inventory over the period \(0 \leq t \leq n_t\) is given

\[
n \int_0^{n_t} \theta(t)\,dt = \frac{nP}{\theta} \left[ t_p + \frac{\left(e^{-\theta t} - 1\right)}{\theta}\right] = \frac{n(P \cdot t_p - q)}{\theta}.
\] (1)

The total deteriorating items over the period \(0 \leq t \leq n_t\) is given

\[n(P \cdot t_p - q).\] (2)

The total cost over the period \(0 \leq t \leq n_t\) is given

\[
TC = S_a + nS_b + h \left[ \frac{n(P \cdot t_p - q)}{\theta} \right] + C_v \left[ n(P \cdot t_p - q) \right].
\]

4.2 The total cost per cycle time \(T\) of retailer

We assume that the demand of retailer is \(D(t) = \alpha + \beta l_b(t)\). The time \(t_q\) and \(t_{\text{b\_max}}\) denote the time of consuming quantity \(q\) and \(I_{\text{b\_max}}\) respectively. The \(I_b(t)\) denote retailer’s inventory level on the shelf at the time \(t\) and inventory level is related to the demand. So the variation of inventory at time \(t\) can be denote as \(\frac{dl_b(t)}{dt} = -D(t) = -\alpha - \beta l_b(t), \ 0 \leq t \leq t_p\).

With the boundary condition \(I_b(0) = q\). We get retailer’s inventory equation over the period \(0 \leq t \leq t_p\) is \(I_b(t) = \frac{\alpha}{\beta + \theta} \left[ e^{(\beta + \theta)t} - 1\right] + qe^{(\beta + \theta)t}, \ 0 \leq t \leq t_p\).

With the boundary condition \(I_b(t_{\text{b\_max}}) = 0\), retailer’s inventory equation over the period \(n_t \leq t \leq t_{\text{b\_max}}\) is \(I_b(t) = \frac{\alpha}{\beta + \theta} \left[ e^{(\beta + \theta)(t_{\text{b\_max}} - t)} - 1\right], \ n_t \leq t \leq t_{\text{b\_max}}\).

With the boundary condition \(I_b(0) = I_{\text{b\_max}}\). We get \(t_{\text{b\_max}}\) which denotes the time
of retailer’s inventory level consume from \( I_{b_{\text{max}}} \) to zero.

\[
I_{b_{\text{max}}} = \frac{\ln \left( 1 + \frac{(\beta + \theta)I_{b_{\text{max}}}}{\alpha} \right)}{\beta + \theta}
\]

4.2.1 The inventory level doesn’t achieve the maxima shelf space of the retailer \( 0 \leq t \leq (n-1)t_p \)

The maxima shelf space can be denoted as follow

\[
I_{b_{\text{max}}} = \frac{\alpha}{\beta + \theta} \left[ e^{-(\beta + \theta)t_p} - 1 \right] \left( 1 - e^{-(\beta + \theta)(n-1)t_p} \right) + q(1 - e^{-(\beta + \theta)t_p})
\]

And the total inventory over the period \( 0 \leq t \leq (n-1)t_p \) can be written as

\[
\frac{ng - I_{b_{\text{max}}} - [(n-1)\alpha t_p]}{\beta + \theta}
\]

Let \( D_m, m = 1, \ldots, (n-1) \) as sales per unit time \( t_p \) then sales over the period \( 0 \leq t \leq t_p \) can be written as

\[
D_t = \int_0^t D(t)dt = \int_0^t (\alpha + \beta I_b(t))dt = \alpha t_p + \beta \frac{\alpha + q\beta - \alpha t_p \beta - e^{-\beta t_p}(\alpha + q\beta)}{\beta^2}
\]

So the total sales over the period \( 0 \leq t \leq (n-1)t_p \) can be written as

\[
nq - I_{b_{\text{max}}}
\]

4.2.2 The inventory models which retailer’s shelf space arrive maxima and supplier finished transfers \( nt_p \leq t \leq t_{b_{\text{max}}} \)

The inventory over the period \( nt_p \leq t \leq t_{b_{\text{max}}} \) can be written as

\[
\int_0^{t_{b_{\text{max}}}} I_b(t)dt = \int_0^{t_{b_{\text{max}}}} \frac{\alpha}{\beta + \theta} e^{(\beta + \theta)(t_{b_{\text{max}}}-t)} - 1 dt = \frac{I_{b_{\text{max}}} - \alpha t_{b_{\text{max}}}}{\beta + \theta}
\]

And the total sales over the period \( nt_p \leq t \leq t_{b_{\text{max}}} \) can be written as
\[ \int_{0}^{t_{\text{max}}} D(t) dt = \alpha \beta t_{\text{max}} - \frac{\alpha \beta t_{\text{max}}}{\beta} = I_{b_{\text{max}}} \]  

Eq. (3) + (5) we can get the total inventory during the replenish cycle time \( T \)

\[ \frac{nq - \alpha [(n-1)t_{p} + t_{\theta_{\text{max}}})]}{\beta + \theta} \]

Eq. (4) + (6) we can get the total sales during the replenish cycle time \( T \)

\[ nq - I_{b_{\text{max}}} + I_{b_{\text{max}}} = nq \]

As the result of section 4.1 and 4.2, we know that the replenish cycle time \( T \)

\[ T = (n-1)t_{p} + t_{\beta_{\text{max}}} \]

Average cost per unit time of supplier and retailer are as follow

\[ TC_{s} = \frac{1}{T} \left[ S_{s} + nS_{b} + h_{s}\left( \frac{nP}{\theta_{t}} \left[ tp + \left( \frac{e^{-\theta_{t}} - 1}{\theta_{t}} \right) \right] + C_{r}\left[ h(P \cdot t_{p} - q) \right] \right) \right] \]

\[ TC_{r} = \frac{1}{T} h_{r}\left( \frac{nq - \alpha [(n-1)t_{p} + t_{\theta_{\text{max}}})]}{\beta + \theta} \right) \]

Average profit per unit time is

\[ R_{r} = \frac{1}{T} P_{r}(nq) \]

The revenue function: total profit-(supplier’s unit cost + retailer’s units cost)

\[ \pi(n, q) = R_{r} - (TC_{s} + TC_{r}) \]
五、Shelf space constraint model

Figure 5.1 display the relationship between supplier and retailer with shelf space constraint. The cycle time $T$ of the supplier and the retailer are the same. We assume that the demand of retailer is $D(t) = \alpha + \beta I_b(t)$. The time $t_q$ and $t_{WB\max}$ denote the time of consuming quantity $q$ and $W_{b\max}$ respectively.

5.1 The total cost per cycle time $T$ of supplier

The calculation methodology of equations below are the same as section 4.1

$$I_v(t) = \frac{P}{\theta_t} \left(1 - e^{-\theta_t t}\right), \quad 0 \leq t \leq t_p \quad \text{and} \quad t_p = -\ln\left(\frac{P - \theta_t q}{P}\right).$$
The total inventory over the period $0 \leq t \leq mtp$ is given

\[ m \int_{0}^{t_{p}} J_{v}(t) dt = \frac{mP}{\theta_{i}} \left[ tp + \frac{\left( e^{-\theta_{i} t_{p}} \right)}{\theta_{i}} - 1 \right] = \frac{m(P \cdot t_{p} - q)}{\theta_{i}} \]  

(7)

The total deteriorating items over the period $0 \leq t \leq mtp$ is given

\[ m(P \cdot t_{p} - q) \]  

(8)

The total inventory over the period $mt_{p} < t \leq (n - m)t_{q}$ is given

\[ Pt_{ip} - (n - m)q \]  

\[ \frac{\theta_{i}}{\theta_{i}} \]  

(9)

The total deteriorating items over the period $mt_{p} < t \leq (n - m)t_{q}$ is given

\[ Pt_{ip} - (n - m)q \]  

(10)

Eq. (7) + (9) we can get the total inventory during the replenish cycle time $T$

\[ P(mt_{p} + t_{ip}) - nq \]  

\[ \frac{\theta_{i}}{\theta_{i}} \]  

Eq. (8) + (10) we can get the total deteriorating items during the replenish cycle time $T$

\[ P(mt_{p} + t_{ip}) - nq \]

The total cost during the replenish cycle time $T$ is

\[ TC_{s} = S_{s} + nS_{b} + h_{i} \left[ \frac{P(mt_{p} + t_{ip}) - nq}{\theta_{i}} \right] + C_{v} \left[ P(mt_{p} + t_{ip}) - nq \right] \]
5.2 The total cost per cycle time $T$ of retailer

The inventory level $I_b(t)$ denote retailer’s inventory level on the shelf at the time $t$ and inventory level is related to the demand. The retailer’s inventory equation over the period $0 \leq t \leq t_p$ is

$$I_b(t) = \frac{\alpha}{\beta + \theta} \left[e^{(\beta + \theta)t} - 1\right] + q e^{(\beta + \theta)t}, \quad 0 \leq t \leq t_p$$

The retailer’s inventory equation over the period $0 \leq t \leq t_q$ is

$$I_b(t) = \frac{\alpha}{\beta + \theta} \left[e^{(\beta + \theta)t} - 1\right] + \left(W_{b_{\text{max}}} - q\right) e^{(\beta + \theta)t}, \quad 0 \leq t \leq t_q$$

$$t_q = \ln \left(\frac{\alpha + (\beta + \theta)W_{b_{\text{max}}}}{\alpha + (\beta + \theta)W_{b_{\text{max}}} - (\beta + \theta)q}\right) \div (\beta + \theta)$$

The retailer’s inventory equation over the period $0 \leq t \leq t_{b_{\text{max}}}$ is

$$I_b(t) = \frac{\alpha}{\beta + \theta} \left[e^{(\beta + \theta)t} - 1\right], \quad 0 \leq t \leq t_{b_{\text{max}}}$$

Figure 5.2: The relation of retailer inventory level and time

The retailer's inventory level on the shelf at the time $t$ and inventory level is related to the demand. The retailer’s inventory equation over the period $0 \leq t \leq t_p$ is

$$I_b(t) = \frac{\alpha}{\beta + \theta} \left[e^{(\beta + \theta)t} - 1\right] + q e^{(\beta + \theta)t}, \quad 0 \leq t \leq t_p$$

The retailer’s inventory equation over the period $0 \leq t \leq t_q$ is

$$I_b(t) = \frac{\alpha}{\beta + \theta} \left[e^{(\beta + \theta)t} - 1\right] + \left(W_{b_{\text{max}}} - q\right) e^{(\beta + \theta)t}, \quad 0 \leq t \leq t_q$$

$$t_q = \ln \left(\frac{\alpha + (\beta + \theta)W_{b_{\text{max}}}}{\alpha + (\beta + \theta)W_{b_{\text{max}}} - (\beta + \theta)q}\right) \div (\beta + \theta)$$

The retailer’s inventory equation over the period $0 \leq t \leq t_{b_{\text{max}}}$ is

$$I_b(t) = \frac{\alpha}{\beta + \theta} \left[e^{(\beta + \theta)t} - 1\right], \quad 0 \leq t \leq t_{b_{\text{max}}}$$
5.2.1 The inventory level doesn’t achieve the shelf space constraint of the retailer \(0 \leq t \leq (m - 1)t_p\)

The total inventory over the period \(0 \leq t \leq (m - 1)t_p\) can be written as

\[
\frac{mq - W_{b_{\text{max}}}}{\beta + \theta} - [(m - 1)\alpha t_p]
\]

(11)

The total sales in \(0 \leq t \leq (m - 1)t_p\) can be written as

\[
mq - W_{b_{\text{max}}}
\]

(12)

5.2.2 The inventory level achieve the shelf space constraint of the retailer and supplier finished transfers \((m - 1)t_p < t \leq (n - m)t_q\)

The inventory over the period \((m - 1)t_p < t \leq (n - m)t_q\) can be written as

\[
\frac{(n - m)q - \alpha t_q}{\beta + \theta}
\]

(13)

And the total sales over the period \((m - 1)t_p < t \leq (n - m)t_q\) can be written as

\[
(n - m) \int_0^t D(t) dt = (n - m)q
\]

(14)

5.2.3 The inventory level achieve the shelf space constraint of the retailer and inventory consume to zero \((n - m)t_q < t \leq t_{\text{wmax}}\)

The inventory over the period \((n - m)t_q < t \leq t_{\text{wmax}}\) can be written as

\[
\frac{W_{b_{\text{max}}}}{\beta + \theta}
\]

(15)
And the total sales over the period \((n - m)t_q < t \leq t_{w_{\text{max}}}\) can be written as

\[
\int_{0}^{t_{w_{\text{max}}}} D(t)\,dt = \alpha t_{w_{\text{max}}} + \beta \frac{W_{b_{\text{max}}} - \alpha t_{w_{\text{max}}}}{\beta} = W_{b_{\text{max}}}
\]

(16)

Eq. (11) + (13) + (15) we can get the total inventory during the replenish cycle time \(T\) as follow

\[
\frac{nq - \alpha[(m-1)t_p + (n-m)t_q + t_{w_{\text{max}}})]}{\beta + \theta}
\]

Eq. (12) + (14) + (16) we can get the total sales during the replenish cycle time \(T\) as follow

\[
mq-W_{b_{\text{max}}} + (n-m)q + W_{b_{\text{max}}}=nq
\]

As the result of section 5.1, 5.2, and 5.3 we know that the replenish cycle time

\[T = (m-1)t_p + (n-m)t_q + t_{w_{\text{max}}}
\]

Average cost per unit time of supplier and retailer are as follow

\[
TC_s = \frac{1}{T} \left\{S_s + nS_b + h_s \left[\frac{P(mt_p + t_{p}) - nq}{\theta} + nS_s + h_s \left[\frac{P(mt_p + t_{p}) - nq}{\theta}ight]\right] + C_s \left[P(mt_p + t_{p}) - nq\right]\right\}
\]

\[
TC_r = \frac{1}{T} h_r \left[\frac{nq - \alpha[(m-1)t_p + (n-m)t_q + t_{w_{\text{max}}})]}{\beta + \theta}\right]
\]

Average profit per unit time is

\[R_r = \frac{1}{T} P_r(nq)
\]

The revenue function is: total profit-(supplier’s unit cost + retailer’s units cost)

\[
\pi(n, q) = R_r - (TC_s + TC_r)
\]

六、 Numerical analysis

In order to illustrate the proposed model, we give the numerical examples as following. Consider a single supplier-single retailer distribution system with the following characteristics: \(P = 250\), \(\alpha = 100\), \(\beta = 0.2\), \(S_b = 500\), \(S_s = 1500\), \(h_r = 15\), \(h_s = 25\), \(P_r = 50\)
6.1 No shelf space constraint model

We use EXCLE to determine the optimal solution and then perform sensitivity analyses. Based on the computational results as shown in Table 6.1, we obtain the following managerial insights:

Table 6.1 optimal solution and sensitivity analyses

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<th>$q$</th>
<th>$n$</th>
<th>$I_{b_{\text{max}}}$</th>
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<td>300.533</td>
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<tr>
<td>$\beta$</td>
<td>0.1</td>
<td>0.336</td>
<td>82.628</td>
<td>4</td>
<td>208.344</td>
</tr>
<tr>
<td></td>
<td>0.22</td>
<td>0.376</td>
<td>92.138</td>
<td>8</td>
<td>324.838</td>
</tr>
<tr>
<td>$P_r$</td>
<td>45</td>
<td>0.363</td>
<td>89.077</td>
<td>5</td>
<td>244.857</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>0.376</td>
<td>92.145</td>
<td>11</td>
<td>389.085</td>
</tr>
</tbody>
</table>

(1). We found that the value of profit $\pi(n, q)$ has high sensitivity to change by varying the parameter $\alpha$, $\beta$ and $P_r$ respectively.

(2). The parameter $\alpha$ indicated the basic demand quantity in replenishment time $T$. It causes without any stock displayed or sales promotion but the loyalty. From Table 6.1 the higher value of $\alpha$ causes higher value of profit. This result implied that the retailer should use some marketing method to increase consumer’s loyalty and interest.

(3). The higher the value of $\beta$ the higher the value of profit. This result indicated “large piles” of goods displayed on the shelf will lead the customer to buy more.

(4). From Table 6.2 and Figure 6.1 we can see although the influence of $n$ on $I_{b_{\text{max}}}$ is positive, but the profit did not rise identically. It means that the appropriate stock display will stimulate customer but stocked too much inventory may tie up retailer’s capital.
Table 6.2 The relation of profit with parameter $n$ and $I_{b_{\text{max}}}$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$q$</th>
<th>$\pi(n,q)$</th>
<th>$I_{b_{\text{max}}}$</th>
<th>$n$</th>
<th>$q$</th>
<th>$\pi(n,q)$</th>
<th>$I_{b_{\text{max}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>156</td>
<td>2365.33</td>
<td>156.000</td>
<td>8</td>
<td>87</td>
<td>3125.03</td>
<td>319.279</td>
</tr>
<tr>
<td>2</td>
<td>123</td>
<td>2858.23</td>
<td>181.905</td>
<td>9</td>
<td>86</td>
<td>3114.72</td>
<td>337.969</td>
</tr>
<tr>
<td>3</td>
<td>109</td>
<td>3020.74</td>
<td>209.544</td>
<td>10</td>
<td>84</td>
<td>3101.97</td>
<td>351.935</td>
</tr>
<tr>
<td>4</td>
<td>101</td>
<td>3090.10</td>
<td>235.439</td>
<td>11</td>
<td>83</td>
<td>3087.74</td>
<td>366.578</td>
</tr>
<tr>
<td>5</td>
<td>96</td>
<td>3120.51</td>
<td>259.780</td>
<td>12</td>
<td>83</td>
<td>3072.64</td>
<td>382.155</td>
</tr>
<tr>
<td>6</td>
<td>92</td>
<td>3131.41</td>
<td>281.038</td>
<td>13</td>
<td>82</td>
<td>3057.09</td>
<td>393.707</td>
</tr>
<tr>
<td>7</td>
<td>89</td>
<td>3131.42</td>
<td>300.533</td>
<td>14</td>
<td>82</td>
<td>3041.30</td>
<td>406.487</td>
</tr>
</tbody>
</table>

Figure 6.1 The relation between profit and parameter $I_{b_{\text{max}}}$

6.2 Shelf space constraint model

In section 6.1 we obtained the maxima shelf space of the retailer $I_{b_{\text{max}}} = 301.353$, so in the space constraint model we assumed the shelf space constraint of the retailer $W_{b_{\text{max}}} = 300$ and we retained the same parameter in section 6.1. The optimal solution and sensitivity analyses are shown in Table 6.2 and we obtain the following managerial insights:

1. We found that the value of profit $\pi(n,q)$ has high sensitivity to change by varying the parameters $\alpha$, $\beta$ and $W_{b_{\text{max}}}$ respectively.

2. Form the result in Table 6.3 the profit of shelf space constraint model is lower than no shelf space constraint model.

3. Table 6.4 and Figure 6.2 showed that when $W_{b_{\text{max}}} > I_{b_{\text{max}}}$ then the shelf space constraint model is equal to the no shelf space constraint model.
(4). The results of varying $\alpha$ and $\beta$ are the same as section 6.1.

Table 6.3 optimal solution and sensitivity analyses

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$t_p$</th>
<th>$q$</th>
<th>$m$</th>
<th>$n$</th>
<th>$I_{b\text{max}}$</th>
<th>$\pi(m, n, q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>0.383</td>
<td>93.942</td>
<td>6</td>
<td>6</td>
<td>300</td>
<td>2740.925</td>
</tr>
<tr>
<td>110</td>
<td>0.351</td>
<td>86.242</td>
<td>8</td>
<td>8</td>
<td>300</td>
<td>3525.047</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.1</td>
<td>0.367</td>
<td>90.141</td>
<td>6</td>
<td>6</td>
<td>300</td>
</tr>
<tr>
<td>0.3</td>
<td>0.377</td>
<td>92.485</td>
<td>8</td>
<td>8</td>
<td>300</td>
<td>3741.251</td>
</tr>
<tr>
<td>$W_{b\text{max}}$</td>
<td>250</td>
<td>0.373</td>
<td>91.437</td>
<td>5</td>
<td>5</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>0.364</td>
<td>89.341</td>
<td>7</td>
<td>7</td>
<td>301.353</td>
</tr>
</tbody>
</table>

Table 6.4 The relation of profit with parameter $m$, $n$, and $I_{b\text{max}}$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$n$</th>
<th>$q$</th>
<th>$\pi(m, n, q)$</th>
<th>$W_{b\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>65.18</td>
<td>2394.48</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>74.86</td>
<td>2873.67</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>83.15</td>
<td>3052.49</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>91.44</td>
<td>3118.49</td>
<td>250</td>
</tr>
<tr>
<td>7</td>
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<td>88.78</td>
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<td>7</td>
<td>7</td>
<td>89.34</td>
<td>3131.43</td>
<td>350</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>89.34</td>
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</tr>
<tr>
<td>7</td>
<td>7</td>
<td>89.34</td>
<td>3131.43</td>
<td>450</td>
</tr>
</tbody>
</table>

Figure 6.2 The relation between profit and parameter $W_{b\text{max}}$
七、Conclusions

This thesis formulates an ordering-transfer inventory model to deal with a consignment inventory in which the retailer’s demand is dependent on the display inventory level subject to the limited shelf space. The product is deteriorating at a constant rate. The current study simultaneously determines the optimal replenishment policy, including the retailer’s ordering quantity and the supplier’s replenishment frequency, to maximize their joint profit in a cycle.

We first consider the case where the retailer has unlimited shelf space, and then another case that the retailer’s space constraint is taken into account. This work provides numerical examples to demonstrate applicability of the proposed models, and also includes the sensitivity analysis of the optimal solution with respect to the parameters. The computation results show some phenomena as follows.

(1) The integrated profit increases as the retailer’s shelf space increases; but the profit remains the same once the space exceeds a ceiling. Because the demand rate is dependent on the inventory level, a retailer may display each of his items in large quantities to generate greater demand.

(2) As the retailer’s shelf space increase, the supplier’s replenishment frequency and lot sizes increase as well. The joint profit also increases, but remains unchanged while retailer’s shelf space reaches a limit.

(3) The higher the multiplier of the customer’s demand causes the higher the joint profit value and transfer frequency. And the deterioration rate has negative impression on the joint profit value.

Regarding the future research, the proposed model can be extended in several ways to allow for quantity discounts or shortages. The demand function can also be generalized as a function of price or be changed to be time varying.
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