The ELECTRE Multicriteria Analysis Approach Based on Intuitionistic Fuzzy Sets

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Abstract—Over the last decades, intuitionistic fuzzy sets have been applied to many different fields, such as logic programming, medical diagnosis, decision making, etc. The purpose of this paper is to develop a new methodology for solving multi-attribute decision-making problems with intuitionistic fuzzy information by using the concept of ELECTRE method. ELECTRE uses the concept of an outranking relationship. We also use TOPSIS method to rank all of the alternatives and to determine the best alternative. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

I. INTRODUCTION

The intuitionistic fuzzy set (IFS) was first introduced by Atanassov in 1986 [1], which is characterized by a membership function and a non-membership function. The IFS generalizes the fuzzy set and introduced by Zadeh in 1965[2], and has been found to be highly useful to deal with vagueness. Over the last decades, IFS has been applied to many different fields, such as logic programming [3], medical diagnosis [4-5], decision making [6-10,10-1-10-4], etc.

Atanassov & Georgiev [3] presented a logic programming system which uses a theory of IFS to model various forms of uncertainty. De et al. [4] has applied in medical diagnosis using the notion of IFS theory. Atanassov et al. [6] presented IF interpretations of the processes of multi-person and of multi-measurement tool with multi-criteria decision making. Hong and Choi [7] provided new functions to measure the degree of accuracy in the grades of membership of each alternative with respect to a set of criteria represented by vague values. Xu and Ronald [8] presented an application of the intuitionistic fuzzy hybrid geometric (IFHG) operator to multiple attribute decision making based on IFS. Z. Xu [9] developed some similarity measures of IFS. He defined the notions of positive ideal IFS and negative ideal IFS and applied the similarity measures to multiple attribute decision making under intuitionistic fuzzy environment. Szmidot and Kacprzyk [10] applied a new measure of similarity to analyze the extent of agreement in a group of experts. The proposed measure takes into account not only a pure distance between intuitionistic fuzzy preferences but also examines if the compared preferences are more similar or more dissimilar to each other. Lin et al. [11] proposed a new method for handling multi-criteria fuzzy decision-making problems based on IFS. It allows the degrees of satisfiability and non-satisfiability of each alternative with respect to a set of criteria to be represented by intuitionistic fuzzy sets, respectively. Chen and Wang [12] presented interval-valued fuzzy permutation (IVFP) methods for solving multi-attribute decision making problems based on interval-valued fuzzy sets. Li et al. [13] developed a new methodology to solve the multi-attribute group decision-making problems with multiple attributes being considered explicitly and both ratings of alternatives on attributes and weight of attributes being expressed using IFS. Boran et al. [14] proposed the technique for order preference by similarity to ideal solution (TOPSIS) method combined with IFS to select appropriate supplier in a group decision making. In this study, also intuitionistic fuzzy weighted averaging (IFWA) operator is used to aggregate all individual decision makers’ opinions for rating importance of criteria and alternatives.

Nevertheless, the literatures show that none of current studies use the concept of ELECTRE method to solve multi-attribute decision-making problems with intuitionistic fuzzy information.

The ELECTRE method is one of the methods of multiple criteria decision making [15]. The multiple criteria decision making (MCDM) models have two classifications: multiple objective decision making (MODM) and multiple attribute decision making (MADM). MODM have decision variable values which are determined in a continuous or integer domain with either an infinitive or a large number of choices, the best of which should satisfy the decision maker’s constraints and preference priorities. MADM on the other hand are generally discrete, have a limited number of alternatives. They require both intra and inter attributes comparisons and involve explicit tradeoff which are appropriate for the problem explained [16].

ELECTRE method was first introduced by Benayoun et al. [15]. The origins of ELECTRE methods go back to 1965 at the European consultancy company SEMA, which is still active today. At that time, a research team from SEMA worked on a concrete, multiple criteria, real-world problem regarding decisions dealing with the development of new activities in firms [17]. The method uses the concept of an ‘outranking relations’. Its first idea concerning concordance, discordance and outranking concepts originated from real-world applications [18]. The method uses concordance and discordance indexes to analyze the outranking relations among the alternatives [19].

In this paper, we develop a new methodology for solving
multi-attribute decision-making problems with intuitionistic fuzzy information by using the concept of ELECTRE method. We also use the TOPSIS index to rank all of the alternatives and to determine the best alternative.

II. INTUITIONISTIC FUZZY SETS

In this section, the concept and operations of IFSs and some of the related distance measures are described.

A. Concept

Let \( X = \{x_1, x_2, ..., x_n\} \) be a finite universal set. An IFS \( A \) in \( X \) is defined as an object of the following form:
\[
A = \{x_j, \mu_A(x_j), \nu_A(x_j) > | x_j \in X \}
\]
where the functions
\[
x_j \in X \rightarrow \mu_A(x_j) \in [0,1],
\]
\[
x_j \in X \rightarrow \nu_A(x_j) \in [0,1].
\]

Define the degree of membership and the degree of non-membership of the element \( x_j \in X \) to the set \( A \subseteq X \), respectively, and for every \( x_j \in X \),
\[
0 \leq \mu_A(x_j) + \nu_A(x_j) \leq 1.
\]
We call
\[
\pi_A(x_j) = 1 - \mu_A(x_j) - \nu_A(x_j)
\]
as the intuitionistic index of the element \( x_j \) in the set \( A \). It is the degree of indeterminacy membership of the element \( x_j \) to the set \( A \).

It is obvious that for every \( x_j \in X \),
\[
0 \leq \pi_A(x_j) \leq 1.
\]

B. Operation

Reference [1], [20] and [21] shows that some operations for IFS. For every two IFSs \( A \) and \( B \) the following operations and relations are valid:

1. \( A \subset B \iff \forall x \in X, (\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \)
2. \( A = B \iff A \subset B \& B \subset A \);   
3. \( \bar{A} = \{(x, \nu_A(x), \mu_A(x))\}. \)

C. Some related distance measures

In many practical and theoretical problems, in order to find the difference between two objects, the knowledge of distance between two fuzzy sets is necessary. Some popular distance measure formula were introduced between two IFSs \( A \) and \( B \) that take into account the membership degree \( \mu \), the non-membership degree \( \nu \), and the hesitation degree (or intuitionistic fuzzy index) \( \pi \) in \( X = \{x_1, x_2, ..., x_n\} \). Some of the intuitionistic fuzzy distance measures are as follows [22-23]:

--Intuitionistic Hamming distance:
\[
dis_1(A,B) = \frac{1}{2n} \sum_{j=1}^{n} (|\mu_A(x_j) - \mu_B(x_j)| + |\nu_A(x_j) - \nu_B(x_j)|).
\]

--Intuitionistic Euclidean distance:
\[
dis_2(A,B) = \left( \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_j) - \mu_B(x_j)|^2 + (\nu_A(x_j) - \nu_B(x_j))^2 \right)^{\frac{1}{2}}.
\]

III. INTUITIONISTIC FUZZY ELECTRE METHOD

A. Solution Process

All solution process in this paper are divided into two parts.

They are IF ELECTRE Method (Step 1 to 8) and Ranking Method (Step 5b to 8b), and all steps shown in Fig. 1.

B. IF ELECTRE Method

The IF ELECTRE method is included in eight steps. The steps are as follows.

Step 1. Determine the decision matrix:

Let \( X_j = (\mu_{ij}, \nu_{ij}, \pi_{ij}) \),
\( \mu_{ij} \) is the degree of membership of the \( i \)th alternative with respect to the \( j \)th attribute, \( \nu_{ij} \) is the degree of non-membership of the \( i \)th alternative with respect to the \( j \)th attribute, \( \pi_{ij} \) is the intuitionistic index of the \( i \)th alternative with respect to the \( j \)th attribute. \( M \) is an intuitionistic fuzzy decision matrix, where
\[
0 \leq \mu_{ij} + \nu_{ij} \leq 1, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n
\]
\[
\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}
\]
In the decision matrix $M$, have $m$ of alternatives (from $A_1$ to $A_m$) and $n$ of attributes (from $X_1$ to $X_n$).

The subjective importance of attributes, $W$, are given by the decision maker(s). For example, attribute $x_1$ has attribute weight $w_1$, $x_n$ has attribute weight $w_n$ and the sum of weight of all attributes from $X_1$ to $X_n$ are equal to 1.

**Step 2.** Determine the concordance and discordance sets:

It use the concept of IFS relation to identify (determine) concordance and discordance set. For example, we can classify different types of the concordance sets as strong concordance set or moderate concordance set or weak concordance set or discordance set. For example, we can classify different types of the concordance sets as strong concordance set, where $\mu \geq \pi$, moderate concordance set, where $\mu > \pi$, and weak concordance set, where $\mu > \pi$.

The strong discordance set $D_{kl}$ of $A_k$ and $A_l$ is composed of all criteria for which $A_k$ is preferred to $A_l$. In other words, the strong discordance set $D_{kl}$ can formulate as

$$D_{kl} = \{ j \mid \mu_{kj} > \mu_{lj} \}.$$  \hspace{1cm} (10)

The moderate discordance set $D_{kl}^m$ is defined as

$$D_{kl}^m = \{ j \mid \mu_{kj} > \mu_{lj} \}.$$  \hspace{1cm} (11)

The weak discordance set $D_{kl}^w$ is defined as

$$D_{kl}^w = \{ j \mid \mu_{kj} > \mu_{lj} \}.$$  \hspace{1cm} (12)

The strong discordance set $D_{kl}$ is composed of all criteria for which $A_k$ is not preferred to $A_l$.

The strong discordance set $D_{kl}$ can formulate as

$$D_{kl} = \{ j \mid \mu_{kj} < \mu_{lj} \}.$$  \hspace{1cm} (13)

The weak discordance set $D_{kl}^m$ is defined as

$$D_{kl}^m = \{ j \mid \mu_{kj} < \mu_{lj} \}.$$  \hspace{1cm} (14)

The weak discordance set $D_{kl}^w$ is defined as

$$D_{kl}^w = \{ j \mid \mu_{kj} < \mu_{lj} \}.$$  \hspace{1cm} (15)

The decision maker(s) give the weight in different sets. For example, the strong discordance set $D_{kl}$ have it own weight weight $w_c$. Weight of $C_{kl}^1$, $C_{kl}^2$, $D_{kl}$, $D_{kl}^m$ and $D_{kl}^w$ are respectively $w_c$, $w_c$, $w_c$, $w_d$ and $w_d$.

**Step 3.** Calculate the concordance matrix:

The relative value of the concordance sets are measured by means of the concordance index. The concordance index is equal to the sum of the weights associated with those criteria and relation which are contained in the concordance sets. Therefore, the concordance index $c_{kl}$ between $A_k$ and $A_l$ is defined as

$$c_{kl} = w_c \times \sum_{j \in D_{kl}} w_j + w_c \times \sum_{j \in D_{kl}^m} w_j + w_c \times \sum_{j \in D_{kl}^w} w_j.$$  \hspace{1cm} (16)

where $w_c$, $w_c$, $w_c$ are weight in different sets and defined in step 2 and $w_j$ are weight of attributes that are also defined in step 1.

**Step 4.** Calculate the discordance matrix:

The discordance index $d_{kl}$ is defined as follows:

$$d_{kl} = \frac{\max_{j \in D_{kl}} w_j \times \text{dis}(X_{ij}, X_{lj})}{\max_{j \in D_{kl}^m} \text{dis}(X_{ij}, X_{lj})}.$$  \hspace{1cm} (17)

where $w_j$ is equal to $w_d$ or $w_c$ or $w_d$ that depend on the different types of discordance sets and defined in step 2.

**Step 5.** Determine the concordance dominance matrix:

This matrix can be calculated with the aid of a threshold value for the concordance index. $A_k$ will only have a chance of dominating $A_l$, if its corresponding concordance index $c_{kl}$ exceeds at least a certain threshold value $\bar{c}$ i.e., $c_{kl} \geq \bar{c}$, and

$$\bar{c} = \frac{\sum_{k=1}^m \sum_{l=1, l \neq k}^m c_{kl}}{m \times (m-1)}.$$  \hspace{1cm} (19)

On the basis of the threshold value, a Boolean matrix $F$ can be constructed, the elements of which are defined as

$$f_{kl} = 1, \text{ if } c_{kl} \geq \bar{c}; f_{kl} = 0, \text{ if } c_{kl} < \bar{c}.$$  \hspace{1cm} (20)

Then each element of 1 on the matrix $F$ represents a dominance of one alternative with respect to another one.

**Step 6.** Determine the discordance dominance matrix:

This matrix is constructed in a way analogous to the $F$ matrix on the basis of a threshold value $\overline{d}$ to the discordance indices. The elements of $G_{kl}$ of the discordance dominance matrix $G$ are calculated as

$$g_{kl} = \frac{\sum_{k=1}^m \sum_{l=1, l \neq k}^m d_{kl}}{m \times (m-1)}.$$  \hspace{1cm} (21)

Again the unit elements in the $G$ matrix represent the dominance relationships between any two alternatives.

**Step 7.** Determine the aggregate dominance matrix:

This step is to calculate the intersection of the concordance dominance matrix $F$ and discordance dominance matrix $G$. The resulting matrix, called the aggregate dominance matrix $E$, is defined by means of its typical elements $e_{kl}$ as:

$$e_{kl} = f_{kl} \times g_{kl}.$$  \hspace{1cm} (22)

**Step 8.** Eliminate the less favorable alternatives:

The aggregate dominance matrix $E$ gives the partial-preferece ordering of the alternatives. If $e_{kl} = 1$, then $A_k$ is preferred to $A_l$ for both the concordance and discordance criteria, but $A_k$ still has the chance of being dominated by the other alternatives. Hence the condition that $A_k$ is not dominated by ELECTRE procedure is,

$$e_{kl} = 1, \text{ for at least one } l, \text{ i.e., } l = 1, 2, \ldots, m, \text{ } k \neq l;$$

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where $e_k = 0$, for all $i \in 1, 2, \ldots, m$, $i \neq k, i \neq l$.

This condition appears difficult to apply, but the dominated alternatives can be easily identified in the $E$ matrix. If any column of the $E$ matrix has at least one element of 1, then this column is “ELECTREcally” dominated by the corresponding row(s). Hence we simply eliminate any column(s) which have an element of 1.

C. Ranking Method with TOPSIS index

Because of the IF ELECTRE Method cannot rank all of the alternatives we utilize TOPSIS index to rank them. Yoon and Hwang [15] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest and Hwang [15] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest and Hwang [15] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest and Hwang [15] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest and Hwang [15] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest and Hwang [15] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest

Step 5b. Determine the concordance dominance matrix: It use the positive-ideal solution of TOPSIS, if $c'$ is the biggest value in the concordance matrix, then calculate

$$c'_{kl} = c^* - c_{kl}$$  \hspace{1cm} (22)

and determine the concordance dominance matrix $C'$.

Step 6b. Determine the discordance dominance matrix $D'$:

Let $d'$ is the biggest value in the discordance matrix, then calculate

$$d'_{kl} = d^* - d_{kl}$$  \hspace{1cm} (23)

and determine the discordance dominance matrix $D'$.

Step 7b. Determine the aggregate dominance matrix $P$:

$$P = \begin{bmatrix}
-p_{11} & \cdots & -p_{1m} \\
-p_{21} & \cdots & -p_{2m} \\
\vdots & \ddots & \vdots \\
-p_{m1} & \cdots & -p_{mm}
\end{bmatrix}$$  \hspace{1cm} (24)

The element $p_{kl}$ of $P$ is define as follows:

$$p_{kl} = \frac{d'_{kl}}{c'_{kl} + d'_{kl}}$$  \hspace{1cm} (25)

where $c'_{kl}$ are the element of concordance dominance matrix and $d'_{kl}$ are the elements of discordance dominance matrix.

Step 8b. Determine the best alternative:

Using the result of step 7b, we can calculate the min evaluation value of alternatives, the formula as below

$$p_k = \frac{1}{m-1} \sum_{j=1}^{m} p_{kj} \quad k = 1, 2, \ldots, m.$$  \hspace{1cm} (26)

Thus, the best alternative $A^*$ can be generated so that

$$A^* = \max \{p_k\}$$  \hspace{1cm} (27)

and the alternatives are ranked according to the increasing order of $A_j$.

IV. Numerical example

In this section, we present a numerical example connected with a decision making problem. Suppose that the proposed ELECTRE method with IFS evaluates the intuitionistic fuzzy decision matrix which refers to 6 of alternatives on 4 of attributes. The intuitionistic fuzzy decision matrix $M$ in step 1 is given.

<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35 0.30 0.32</td>
<td>0.22 0.40 0.34</td>
<td>0.23 0.59 0.18</td>
<td>0.10 0.87 0.03</td>
<td></td>
</tr>
<tr>
<td>0.24 0.34 0.42</td>
<td>0.26 0.42 0.32</td>
<td>0.31 0.28 0.41</td>
<td>0.44 0.39 0.17</td>
<td></td>
</tr>
<tr>
<td>$M = A_1$</td>
<td>0.11 0.10 0.73</td>
<td>0.19 0.47 0.34</td>
<td>0.11 0.34 0.55</td>
<td>0.19 0.66 0.15</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.09 0.38 0.53</td>
<td>0.43 0.29 0.28</td>
<td>0.44 0.24 0.32</td>
<td>0.41 0.26 0.33</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.37 0.56 0.07</td>
<td>0.34 0.39 0.27</td>
<td>0.37 0.35 0.28</td>
<td>0.34 0.25 0.41</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.25 0.34 0.41</td>
<td>0.29 0.35 0.36</td>
<td>0.30 0.28 0.42</td>
<td>0.45 0.48 0.07</td>
</tr>
</tbody>
</table>

Assume that the subjective importance of attributes, $W$, is given by the decision maker,

$W = [w_1, w_2, w_3, w_4] = [0.1, 0.2, 0.3, 0.4]$. Applying step 2, determine the concordance and discordance sets.

The decision maker also give the relative weight ($W'$).

$W' = [w_c, w_c, w_c, w_d, w_d, w_d, w_d, w_d] = [1, 2, 1, 3, 3, 3, 3]$. The strong concordance set

$$C_{kl} = \begin{bmatrix}
- & 1 & - & - & - \\
2 & - & 2,3 & 1 & - \\
- & - & - & - & - \\
2 & 2 & 3 & - & - \\
- & - & 3,4 & - & - \\
\end{bmatrix}$$

For example, $C_{21} = \{2\}$, which is in the 2 nd (horizontal) row and 1 st (vertical) column of strong concordance set is “2”, $C_{13} = \{-\}$, which is in the 1 st row and 3 th column of strong concordance set is “empty”, and so forth.

The moderate concordance set

$$C_{kl} = \begin{bmatrix}
- & - & 2 & - & - & - \\
3,4 & - & 4 & - & - & - \\
4 & - & - & - & - & - \\
- & - & 3,4 & - & - & - \\
- & - & 2,3,4 & 2 & 2 & - & - \\
- & - & 2,3 & 2 & 2 & - & - \\
\end{bmatrix}$$

The weak concordance set

$$C'_{kl} = \begin{bmatrix}
- & - & 1,3 & - & - & - \\
- & - & 1 & 4 & 4 & 3 \\
- & - & - & - & - & - \\
1 & 1,3 & 13 & - & 1,2,3 \\
- & - & 1,4 & 1 & 4 & 4 \\
\end{bmatrix}$$

The strong discordance set

$$D_{kl} = \begin{bmatrix}
- & - & - & 2 & 2 & - \\
1 & - & - & - & - & - \\
2 & 2,3 & - & 2,3 & 2 & 3,4 \\
1 & 1 & - & - & - & 1 \\
- & - & - & - & - & - \\
1 & - & - & - & - & - \\
\end{bmatrix}$$

The moderate discordance set
renders the following overranking

Applying step 3, calculate the concordance matrix is.

The weak discordance set

Applying step 4, calculate the discordance matrix is.

and

Applying step 5, determine the discordance dominance matrix is. The average discordance index is

The concordance dominance matrix is

The average concordance index is

Applying step 6, determine the discordance dominance matrix is. The average discordance index is

Applying step 7, determine the aggregate dominance matrix (E) is.

Applying step 8, eliminate the less favorable alternatives is.

The matrix E renders the following overranking relationships: $A_2 \rightarrow A_1, A_2 \rightarrow A_3, A_4 \rightarrow A_3, A_4 \rightarrow A_5, A_5 \rightarrow A_1, A_3 \rightarrow A_5, A_3 \rightarrow A_6, A_6 \rightarrow A_4$ (illustrated with Fig. 2).
is\(c^* = 0.8667\).

The concordance outranking matrix is:
\[
C = \begin{bmatrix}
-0.7667 & 0.6 & 0.7667 & 0.8667 & 0.7667 \\
0.2 & -0.0667 & 0.6334 & 0.7334 & 0.7667 \\
0.6 & 0.8667 & -0.8 & 0.8667 & 0.8887 \\
0.2 & 0.3667 & 0.1 & -0.4 & 0.3667 \\
0.1667 & 0.5334 & 0.2667 & 0.8334 & -0.6667 \\
0.2667 & 0.5667 & 0 & 0.6334 & 0.7334
\end{bmatrix}
\]

Applying step 6b, calculate discordance outranking matrix is\(d^* = 1\).

The concordance outranking matrix is:
\[
D = \begin{bmatrix}
-0.3333 & 0.6590 & 0.3333 & 0.3333 & 0.3333 \\
0.7536 & -1 & 0 & 0.6667 & 0.5749 \\
0.6667 & 0.3407 & -0.1553 & 0.5840 & 0.1699 \\
0.5477 & 0.1261 & 0.5952 & -0.6667 & 0.4052 \\
1 & 0.7729 & 1 & 0.8399 & -0.6667 \\
0.7433 & 0.8952 & 1 & 0.4849 & 0.6687
\end{bmatrix}
\]

Applying step 7b, determine the aggregate outranking matrix is.
\[
P = \begin{bmatrix}
-0.303 & 0.5234 & 0.303 & 0.2778 & 0.303 \\
0.7903 & -0.9375 & 0 & 0.4762 & 0.4285 \\
0.5263 & 0.2822 & -0.1626 & 0.4026 & 0.1639 \\
0.7325 & 0.2559 & 0.8562 & -0.6250 & 0.5249 \\
0.8571 & 0.5917 & 0.7895 & 0.5019 & -0.5 \\
0.7359 & 0.6124 & 1 & 0.4336 & 0.4769
\end{bmatrix}
\]

Applying step 8b, determine the best alternative is.
\[
\bar{p}_1 = 0.342, \bar{p}_2 = 0.5265, \bar{p}_3 = 0.3075, \bar{p}_4 = 0.5989, \bar{p}_5 = 0.648, \bar{p}_6 = 0.6518.
\]

The optimal ranking order of the alternatives is given by \(A_6 \succ A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2\). The best alternative is \(A_6\).

V. CONCLUDING REMARKS

In this study, we have provided a new methodology for solving multi-attribute decision-making problems with intuitionistic fuzzy information by using the concept of ELECTRE method. The new approach integrate the concept of ‘outranking relationship’ of ELECTRE method. We also used the Ranking method with TOPSIS index to rank all of the alternatives and to determine the best alternative. We also illustrated numerical example to demonstrate its practicality and effectiveness. In a future research, we shall utilize the concept of interval-valued intuitionistic fuzzy sets to develop the multiple criteria decision making methods.

REFERENCES


